



The In-Transit Vigilant Covering Tour Problem for
Routing Unmanned Ground Vehicles

THESIS

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THE IN-TRANSIT VIGILANT COVERING TOUR PROBLEM FOR ROUTING
UNMANNED GROUND VEHICLES

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Abstract

The routing of unmanned ground vehicles for the surveillance and protection of key installations is modeled as a new variant of the Covering Tour Problem (CTP). The CTP structure provides both the routing and target sensing components of the installation protection problem. Our variant is called the in-transit Vigilant Covering Tour Problem (VCTP) and considers not only the vertex cover but also the additional edge coverage capability of the unmanned ground vehicle while sensing in-transit between vertices. The VCTP is formulated as a Traveling Salesman Problem (TSP) with a dual set covering structure involving vertices and edges. An empirical study compares the performance of the VCTP against the CTP on test problems modified from standard benchmark TSP problems to apply to the VCTP. The VCTP performed generally better with shorter tour lengths but at higher computational cost.

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THE IN-TRANSIT VIGILANT COVERING TOUR PROBLEM FOR ROUTING UNMANNED GROUND VEHICLES

I. Introduction

1.1 Overview

Operations research techniques are frequently applied to the entire spectrum of military scenarios and are used to identify optimal usage of scarce resources. One of the key techniques is the application of combinatorial optimization models as decision support models. These models are useful for the analysis of complex military scenarios and provide military commanders with a quantitative basis for the evaluation of decision options. Some of these scenarios include aircraft scheduling, logistics delivery, routing and target coverage.

Within the Air Force Institute of Technology (AFIT), the Maximization of Observability in Navigation for Autonomous Robotic Control (MONARC) project has an overarching goal to develop an autonomous robotic, network-enabled, Search, Track, ID, Geo-locate, and Destroy (Kill Chain) capability which would be effective in any environment, at any time. One of the specific mission scenarios for MONARC is mission planning for routing Unmanned Ground Vehicles (UGVs) for base security.

Combinatorial optimization approaches on related problems such as routing Unmanned Aerial Vehicles (UAVs) have been proposed. Ryan *et al.* [1998] discussed a multiple Traveling Salesman Problem with time windows (mTSPTW) formulation with the objective of maximizing target coverage. A reactive tabu search heuristic was applied

to solve routing problems for UAVs reconnaissance. The Vehicle Routing Problem with time window (VRPTW) approach, with the application of Java-encoded metaheuristic, was used [O'Rourke *et al.*, 2001] for the dynamic routing of UAVs. Harder *et al.* [2004] added new UAV considerations and tabu search techniques, and proposed a layered architecture to support pre-planning and real-time tasking of UAVs.

The MONARC security defense task requires the consolidation of intelligence, management of system readiness, centralized operational planning and dissemination of Command and Control (C2) information for the provision of a surveillance approach for use by the team of UGVs. This surveillance approach requires the UGVs to perform their surveillance functions by visiting multiple locations, while covering some locations, in the shortest tour route possible. A new variant of the multiple-vehicle Covering Tour Problem (mCTP) was developed and evaluated to model this surveillance approach. The mCTP consists of determining a set of total minimum length vehicle routes on a subset of V , subject to side constraints, such that every vertex of W is within a pre-determined distance from a route [Hachicha *et al.*, 2000].

1.2 Research Focus

This research presents the development of a new variant of the Covering Tour Problem (CTP), which considers target coverage by both vertices and edges, to model a generic base security defense scenario. The CTP consists of determining a minimum length Hamiltonian cycle on a subset of V such that every vertex of W is within a pre-determined distance from the cycle [Gendreau *et al.*, 1997]. As the UGVs are able to sense while traveling, the CTP model, which considers coverage only at vertices, is

artificially limiting. An empirical study is designed and conducted to examine the benefits and costs of the new variant of the CTP.

The new variant CTP model is formulated as a TSP with dual set covering structure involving vertices and edges. It is coded in LINGO 11.0 and tested on various sets of randomly generated problems in comparison with the CTP model. The optimal tour length, type of coverage and computational effort are recorded and compared.

1.3 Thesis Organization

Chapter 2 of this thesis provides background of the base security problem. The CTP model is described in detail along with other applications of the model. The relationship of other combinatorial optimization problems with the CTP completes the chapter. Chapter 3 is written as a journal article and defines the new variant of the CTP model as the in-transit Vigilant CTP (VCTP). The mathematical formulation, methodology for empirical study and results concludes the chapter. Chapter 4 contains a discussion of the conclusions and recommended future research areas.

II. Related Work

The Covering Tour Problem (CTP) is a combinatorial optimization problem that can be applied to many military scenarios. Such scenarios include the placement of multiple UAVs for perimeter surveillance [Kinney Jr *et al.*, 2005] in which the area of interest is monitored in a decentralized but optimized fashion. For this thesis, we will focus on the application of the CTP on the security defense task of critical facilities.

The Maximization of Observability in Navigation for Autonomous Robotic Control (MONARC) project within AFIT has an overarching goal of the development of an autonomous robotic, network-enabled, Search, Track, ID, Geo-locate, and Destroy (Kill Chain) capability which would be effective in any environment, at any time. This long-term goal is dependent on the development of novel ways to automate, shorten, and enhance kill-chain effectiveness through higher levels of guidance, navigation, control, and estimation integration, from the sub-system/sensor level all the way up to the operational level using autonomous robotic vehicles.

One of the research areas in the MONARC project is mission planning for base security. This task requires collecting sensory data sampling the environment at different locations, exchange the information with other nodes, and collaboratively accomplish the required mission. The coordination and control of multiple mobile sensors provides an opportunity to improve the quality and robustness of the collected data, as compared to a single sensor and/or static system.

Specifically, the security defense task of critical facilities can be formulated as a CTP for multiple vehicles. With the amalgamation of various sensory inputs into a

Recognized Ground Situation Picture (RGSP), the locations of all security entities and adversaries are known at a specific point in time. As such, it is possible to develop a CTP model, in which multiple security entities (vehicles) must visit multiple adversaries (locations), while covering certain locations, in the shortest distance possible travelled by all entities.

2.1 Background of base security problem

One of the defined MONARC scenarios relates to the protection of Key Installations (KIN) from potential adversarial intrusions. A team of Unmanned Ground Vehicles (UGVs) are tasked to protect a critical installation. Their surveillance capabilities are augmented by static sensors that are located throughout the installation. Therefore, a RGSP is available to a mission planner to assist in finding UGV tour routes. The UGVs should only patrol routes that cover all the required checkpoints and the overall route length should be minimized. All UGVs originate from a base station, the depot. There are certain checkpoints that the UGVs must visit (these checkpoints are usually critical ones requiring compulsory surveillance) and there are also checkpoints that may be visited. There are also potential spots where the adversary may appear and these spots must be covered by visiting a checkpoint that is within a fixed proximity distance. Once, each checkpoint is visited by a UGV, all UGVs return to the base station.

The critical installation protected by the UGV team is modeled as a complete graph with a vertex set as the various checkpoints to visit. The graph is undirected; UGVs travel either direction. Within the vertex set, there may be predefined critical checkpoints that must be visited. The potential adversarial spots (targets) are modeled using a second

vertex set, all of which must be covered by a UGV tour. This second vertex set is excluded from the tour route construction to prevent any UGV from traveling directly into an adversary.

Coverage of targets by a visited checkpoint is defined as the circular area of a fixed radius, where any vertex within the area is covered by that checkpoint. The circular area of coverage is analogous to the effective range of a weapon or sensor system on-board the UGVs. Each UGV is modeled as an individual vehicle travelling on different routes of minimum length tours. During the route, the UGV covers targets and all targets must be covered for a feasible solution to the overall problem.

There are some key assumptions and limitations made in modeling the base defense security scenario as a multiple CTP:

- a. All UGVs are similar and have equal capabilities of movement and coverage.
- b. UGVs can visit as many vertices and as long a tour length as required.
- c. UGVs travel in straight lines between vertices.
- d. Potential adversarial spots are known at a specific point of time or are pre-defined and are thus part of the problem structure.

2.2 Other applications of CTP

One of the main applications of the CTP lies in the health care industry, especially for the deployment of mobile health care units in developing countries [Hodgson *et al.*, 1998]. The mobile health care units have access to a limited number of villages due to factors such as infrastructure restrictions, unit capacity and cost. Therefore, it is not feasible to travel to all villages. Instead, a tour route is planned so that the unvisited

villages are within reasonable walking distance to the visited villages for the needy to receive health care. By modeling this as a CTP, the vehicle routes of the health care units are efficiently planned to reduce the amount of travel required, yet provide sufficient medical coverage. A real-life problem associated with the planning of mobile health care units in the Suhum district, Ghana, was studied [Oppong *et al.*, 1994] and solved [Hachicha *et al.*, 2000] as a CTP.

Another important application of the CTP is the placement of post box locations to reduce the traveling distance of the postal delivery service while ensuring maximum coverage [Labbé *et al.*, 1986]. Good locations of post boxes to cover a region of users and an optimal route for collection are constructed. Alternatively, this can also be applied to the management of centralized post offices, i.e. post offices are centralized at towns of higher populations and the smaller towns nearby are covered by the centralized post offices.

The CTP model can be applied to the transportation industry such as the design of bi-level, hierarchical transportation networks [Current *et al.*, 1994]. For an overnight mail delivery service provider such as DHL or Fedex, the optimal tour route represents the route taken by the primary vehicle (aircraft) to the distribution centers and the coverage radius is represented by the maximum distance travelled by the delivery trucks from the distribution centers to its customers. This ensures that the distribution cost to every region is minimized while providing the required delivery service to customers. Drilling into the problem further, the CTP does not consider the efficient distribution by the delivery trucks. However, this could be solved by considering each distribution center as a TSP.

The design of computer networks can also be modeled as a CTP with the objective of minimizing the connection cost of numerous computers to the nearest servers. The servers are then modeled as the vertices with the computers as the vertices to be covered.

2.3 Explanation of CTP

The CTP is classified as a NP-hard problem [Garey *et al.*, 1979] as it reduces to a Traveling Salesman Problem (TSP) when the coverage distance is zero and all the potential adversarial spots must be visited rather than covered.

The multi-vehicle variant of the CTP (mCTP) is defined [Hachicha *et al.*, 2000] as a complete undirected graph $G = (V \cup W, E)$ where $V \cup W$ is the vertex set, and $E = \{(v_i, v_j) | v_i, v_j \in V \cup W, i < j\}$ is the edge set. Vertex v_0 is a depot (base station), V is the set of vertices that can be visited, $T \subseteq V$ is the set of vertices that must be visited ($v_0 \in T$), and W , the set of vertices (or targets) that must be covered. A distance matrix $C = (c_{ij})$, which satisfies the triangle inequality, indicates the edge length between all vertices ($V \cup W$) is defined for E . A final parameter is c , the pre-defined maximum size of the cover.

The solution of the mCTP consists in defining a set of m vehicle routes of minimum total length satisfying the following constraints:

1. Each vehicle route starts and ends at the base station, v_0 , subject to a maximum of m vehicle routes.
2. Each vertex of V belongs to at most one route and each vertex of T belongs to exactly one route.

3. Each vertex of W must be covered by a route, i.e. it lies within a distance c of a vertex V which belongs to a route. Additionally, the depot should not cover all vertices in W .

The mCTP can be formulated as a linear integer problem. For $v_h \in V$, let y_{hk} be a binary variable equal to 1 if and only if vertex v_h is visited by vehicle k and belongs to the tour; otherwise, y_{hk} is zero. If $v_h \in T$, then y_{hk} is equal to 1.

For $v_i, v_j \in V$ and $i < j$, let x_{ijk} be a binary variable equal to 1 if and only if edge (v_i, v_j) belongs to the tour and is travelled by vehicle k . However, for the special case of $i = 0$ (for route originating from depot), then x_{ijk} can take values of 0, 1 and 2 (for the returning trip). Otherwise, x_{ijk} is zero.

For every $v_l \in W$, we define a covering set $S_l = \{v_h \in V \mid c_{hl} \leq c\}$ which detects all vertices of the set V that is able to cover the vertex $v_l \in W$. Thus, there should be vertices $v_h \in V$ which lie at a distance c_{hl} from $v_l \in W$, where c_{hl} is less than or equal to the predetermined covering distance c .

The formulation of the mCTP, which minimizes the tour length, is as follows:

$$\text{Minimize } \sum_{k=1}^m \sum_{i=0}^{n-1} \sum_{j=i+1}^n c_{ij} x_{ijk} \quad (1)$$

Subject to

$$\sum_{k=1}^m \sum_{v_h \in S_l} y_{hk} \geq 1 \quad (\forall v_l \in W) \quad (2)$$

$$\sum_{k=1}^m y_{hk} \leq 1 \quad (\forall v_h \in V \setminus \{v_0\}) \quad (3)$$

$$\sum_{i=0}^{h-1} x_{ihk} + \sum_{j=h+1}^n x_{hjk} = 2y_{hk} \quad (\forall v_h \in V \setminus \{v_0\}, k = 1, 2, \dots, m) \quad (4)$$

$$\sum_{k=1}^m \sum_{v_i \in S, v_j \in V \setminus S} x_{ijk} \geq 2 \sum_{k=1}^m y_{hk} \quad (S \subset V, T \setminus S \neq \emptyset, v_h \in S) \quad (5)$$

$$\sum_{j=1}^n x_{0jk} \leq 2 \quad (\forall k = 1, 2, \dots, m) \quad (6)$$

$$y_{hk} = 1 \quad (\forall v_h \in T, k = 1, 2, \dots, m) \quad (7)$$

$$y_{hk} \in \{0, 1\} \quad (\forall v_h \in V \setminus T, k = 1, 2, \dots, m) \quad (8)$$

$$x_{0jk} \in \{0, 1, 2\} \quad (k = 1, 2, \dots, m) \quad (9)$$

$$x_{ijk} \in \{0, 1\} \quad (k = 1, 2, \dots, m) \quad (10)$$

Constraint (2) ensures that all vertices $v_l \in W$ are covered by at least one vertex. Constraint (3) ensures that each vertex $v_h \in V$, except v_0 , is visited at most once during the tour. Constraint (4) is the degree constraint and ensures that if vertex $v_h \in V$ is visited by vehicle k , then there will be an entering and exiting edge. Constraint (5) is the connectivity constraint which eliminates subtours. It ensures that for every subset S of V , there are at least 2 edges that connect a set S and the complementary set $V \setminus S$. Constraint (6) ensures that for each vehicle k leaving the depot, there is a maximum of 2 edges for entering and leaving the depot. Constraint (7), (8), (9) and (10) are the binary and integer constraints. Specifically, constraint (7) ensures that vertex $v_h \in T$ must be visited once. Constraint (8) is a binary variable which equal to 1 if and only if vertex v_h is visited by vehicle k and belongs to the tour. Constraint (9) and (10) are to ensure that if only if edge (v_i, v_j) belongs to the tour and is travelled by vehicle k , x_{ijk} takes value of 1.

The CTP was first introduced in 1981 [Current] and formulated in 1989 [Current and Schilling]. It was formulated as a linear integer program in 1995 [Gendreau *et al.*]. There are many variants of the CTP for different applications and a few of them are mentioned below.

1. The mCTP is a natural extension of the single vehicle CTP, which is a generalization of the CTP. The objective is to design m Hamiltonian cycles over a subset of eligible vertices in the vertex set V to visit or cover all of the vertices in the complete undirected graph G . Three heuristics solution approaches were developed.
2. The Generalized CTP (GCTP) was introduced by Motta *et al.* [2001]. It is another generalization of the CTP and consists of finding a minimum length Hamiltonian cycle over a subset of vertices in both vertex sets V and W , rather than exclusively in the vertex set V . A metaheuristic algorithm which follows the Greedy Randomized Adaptive Search Procedures (GRASP) [Feo *et al.*, 1995] was proposed to solve the problem.
3. A bi-objective variant of the CTP was discussed by Jozefowicz *et al.* [2007]. In this generalization, the constraints linked to coverage are replaced by a second objective. Thus, the problem seeks to minimize both the two conflicting objectives; tour length and the coverage distance via a multi-objective evolutionary algorithm. This approach avoids a priori parameterization of the problem rather than working with a family of related problems in which only the covering distance varies.
4. Another multi-objective CTP for disaster relief operation planning was explored by Nolz *et al.* [2010] in which the demand of each node has to be satisfied by exactly one vehicle. It considers the minimization of three objectives: (1) the sum of distances between all nodes and their nearest facility, (2) the total tour length, and (3) the latest

arrival time at a node. As the first objective is in conflict with the second and third objectives, a bi-objective problem is solved by considering objectives 1 and 2 and then objectives 1 and 3.

5. A CTP approach for the location of satellite distribution centers to supply humanitarian aid was proposed by Naji-Azimi *et al.* [2011]. This problem extends the multi-vehicle CTP to include multiple commodities, heterogeneous capacitated fleet and split deliveries.

2.4 Relationship of CTP to other NP problems

The CTP is related to the family of NP-hard problems, such as the Traveling Salesman Problem (TSP), Vehicle Routing Problem (VRP), and Covering Salesman Problem (CSP) etc. We will review some of these important classical problems and highlight the differences and relationship between them.

2.4.1 Traveling Salesman Problem

The Traveling Salesman Problem is one of the classic problems in Operations Research and a well-studied combinatorial optimization problem [Chen *et al.*, 2010]. The TSP is hard to solve both theoretically and in practice. Solving the TSP has thus motivated a variety of solution algorithms including simple heuristics and nature-inspired meta-heuristics.

Given a graph $G = (N, E)$, of node set N and arc set E , the objective is to minimize the tour length of the traveling salesman. Starting from a node, a route is constructed through all nodes in N uniquely and returned to the originating node. From

the seminal paper published in 1954 [Dantzig *et al.*], the TSP has been extensively studied and much literature on TSP theories, formulations, applications and algorithms have been published. One of the earliest integer linear programming formulations by Dantzig *et al.* associates a binary variable x_{ij} to every arc (i, j) , equal to 1 if and only if (i, j) is in the optimal route, $i \neq j$. The formulation is:

$$\text{Minimize } \sum_{(i,j) \in E} c_{ij} x_{ij} \quad (1)$$

Subject to

$$\sum_{j=1} x_{ij} = 1 \quad \forall i = 1, \dots, n \quad (2)$$

$$\sum_{i=1} x_{ij} = 1 \quad \forall j = 1, \dots, n \quad (3)$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1, \quad S \subset N, \quad 2 \leq |S| \leq |N| - 1 \quad (4)$$

The objective function (1) minimizes the optimal tour cost. Constraint (2) and (3) are degree constraints which specify that every vertex is entered once. Constraint (4) presents the subtour elimination constraints.

The TSP naturally arises as a subproblem in many transportation and logistics applications, for example the problem of arranging school bus routes to pick up the children in a school district. This bus application is of important historical significance to the TSP, since it provided motivation for Merrill Flood, one of the pioneers of TSP research in the 1940s. More recent applications involve the scheduling of service calls at cable firms, the delivery of meals to homebound persons, the scheduling of stacker cranes in warehouses, the routing of trucks for parcel post pickup, etc [Applegate *et al.*, 2011].

Although transportation applications are the most natural setting for the TSP, the simplicity of the model has led to many interesting applications in other areas. A classic example is the scheduling of a machine to drill holes in a circuit board or other object. In this case the holes to be drilled are the cities, and the cost of travel is the time it takes to move the drill head from one hole to the next. The technology for drilling varies from one industry to another, but whenever the travel time of the drilling device is a significant portion of the overall manufacturing process then the TSP can play a role in reducing costs.

A major assumption of the TSP is that the salesman must uniquely visit every vertex on the graph. However, this assumption can be relaxed for many real world problems [Current *et al.*, 1989]. Rural health care delivery and aircraft routing are examples in which all villages or cities do not need to be visited as long as they can be covered within a pre-determined distance from the visit point.

The TSP has received a lot of research and study over the years; however it is more appropriate to model some real-world applications as a multiple TSP (mTSP). Many applications of the mTSP were discussed [Bektas, 2006], such as print scheduling, workforce planning, transportation planning, production planning and satellite systems. The mTSP model was used for the mission planning of autonomous mobile robots in various environments [Brummit *et al.*, 1996][Zhong *et al.*, 2002]. The mTSP is similar to the TSP and in general be defined as follows: Given a set of nodes, let there be m salesmen located at a single depot node. The remaining nodes that are to be visited are called intermediate nodes. Then, the mTSP consists of finding tours for all m salesmen, who all start and end at the depot, such that each intermediate node is visited exactly once

and the total cost of visiting all nodes is minimized. The cost metric can be defined in terms of distance, time, etc.

A diagram of the solution for an mTSP with 15 nodes, inclusive of the depot, is shown below.

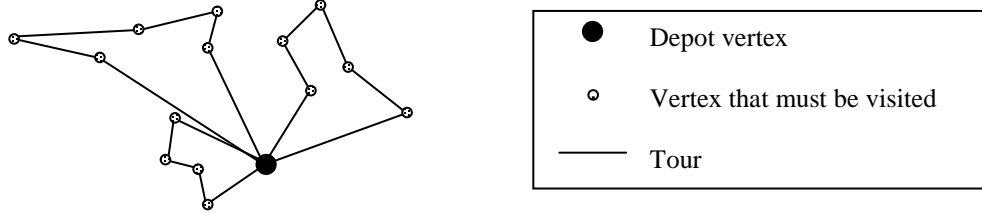


Figure 1: Solution for an mTSP.

In Gendreau *et al.* [1995], the CTP is related to many different variants of the TSP, such as the Prize Collecting Traveling Salesman Problem (PTSP), Selective Traveling Salesman Problem (STSP) and Generalized Traveling Salesman Problem (GTSP) by Fischetti *et al.* [1997].

The GTSP is a version of the classical TSP, in which the set of nodes N has been partitioned into clusters (not necessarily disjointed) of nodes, and the problem is to find a least cost tour that passes through exactly one node from each cluster. Thus, we can express the CTP as a GTSP by defining some nodes into sets. For each node in W (targets to be covered), we define a set in V which covers it. Also, for each node in T (nodes that must be visited), we define it as an individual set. Thus, we can solve the CTP by solving a GTSP on all the defined sets.

2.4.2 Vehicle Routing Problem

The Vehicle Routing Problem (VRP) is to design optimal delivery or collection routes from one or several depots to a number of geographically scattered cities or customers, subject to side constraints. It is one of the most important and well studied combinatorial optimization problems and plays an important problem in the fields of transportation, distribution and logistics.

The VRP was proposed in 1959 [Dantzig *et al.*]. They described a real-world application regarding the delivery of gasoline to service stations and proposed the first mathematical programming formulation. Since then, hundreds of models and algorithms had been proposed for the optimal and approximate solution of different VRP versions. The interest in the VRP is motivated by its practical relevance and its considerable difficulty.

The simplest and most studied of the VRP family is the capacitated VRP. Given a graph $G = (V, A)$, where $V = \{1, \dots, n\}$ is a set of vertices representing cities with the depot located at vertex 1, and A is the set of arcs. A distance matrix $C = (c_{ij})$ is associated with every arc (i, j) and $i \neq j$. In addition, there are m available vehicles based at the depot which are identical and have the same capacity D . The VRP consist of designing a set of least-cost vehicle routes such that each city is visited once by exactly one vehicle and all vehicle routes start and end at the depot. Some common side constraints include [Laporte, 1992]:

1. Capacity restrictions: A non-negative weight d_i is attached to each city and the sum of weights of any vehicle route may not exceed the vehicle capacity.
2. Number of cities on any route is bounded above by q .

3. Total time restrictions: The length of any route may not exceed a prescribed bound L .
4. Time windows: City i must be visited within the time interval $[a_i, b_i]$ and waiting is allowed at city i .
5. Precedence relations between pairs of cities.

The VRP is related to the mTSP. If the VRP has m number of vehicles with capacity constraints removed and all cities have only unit demands, it reduces to an mTSP. Similarly, the mCTP reduces to a VRP when all cities must be visited and have only unit demands.

An example of a classic VRP is shown below.

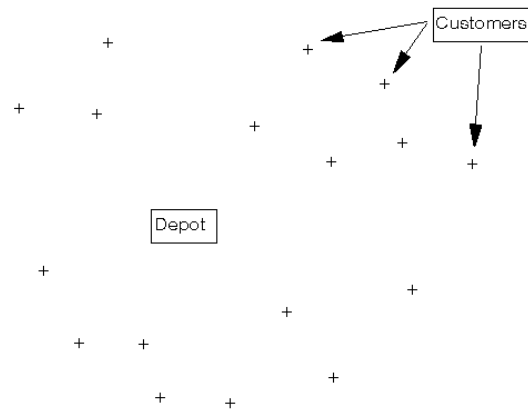


Figure 2: A classic VRP [Beasley, 2012].

Figure 2 shows the situation in which a depot is surrounded by a number of customers who are to be supplied from the depot. The routes for the vehicles (with known capacities) are designed to minimize the total distance traveled while supplying the customers with known demands. The designed routes for the delivery vehicles are shown in Figure 3.

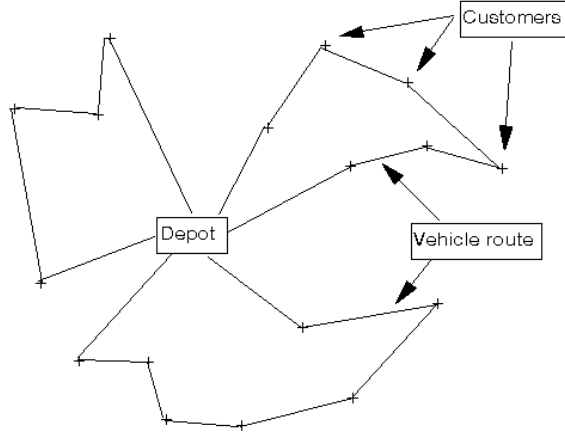


Figure 3: Solution for a VRP [Beasley, 2012].

2.4.3 Covering Salesman Problem

A variant of the TSP was introduced by Current [1981] as the Covering Salesman Problem (CSP). The CSP is similar to the TSP, except that not all nodes need to be visited. The objective is to minimize the tour length of a subset of N number of nodes. For the nodes that are not on the tour, they must be within a pre-determined covering distance c of a node on the tour. Thus, the tour must cover each of the nodes rather than visit it directly. This problem may be considered as a generalization of the TSP. If the covering distance is zero ($c = 0$), each node must be visited directly to be covered. Thus, the CSP reduces to a TSP and is consequently NP-hard.

The CSP was solved by constructing the optimal TSP tour over the minimum number of vertices for a feasible solution. This effectively solves the corresponding Set Covering Problem (SCP). As the associated SCP may have multiple optimal solutions with the same number of vertices, the minimum length tour is found by applying a TSP solver over all the optimal solutions of the SCP.

Two variants of the CSP, known as the Median Tour Problem (MTP) and Maximal Covering Tour Problem (MCTP) were introduced by Current *et al.* [1994]. Given a network of n nodes, both the MTP and MCTP seek to minimize the total tour length over a predefined number of p nodes (where $p \leq n$) and maximize accessibility of the $(n - p)$ covered nodes. In the MCTP, a node is covered if and only if it lies within a predefined distance from a tour node. In the MTP, the accessibility objective is to minimize the total demand multiplied by the travel distance that the covered nodes must traverse to reach their nearest tour node.

The Covering Tour Problem has a close relationship with the CSP and can be considered as a generalization of the CSP [Golden *et al.*, 2011]. The key distinction of the CTP is that some subset of the nodes must be on the tour while the remaining nodes need not be on the tour. Similar to the CSP, a node not on the tour must be within a predefined covering distance of a node on the tour. The CTP reduces to the CSP if the subset of nodes that must be on the tour is empty. Furthermore, the CTP reduces to the TSP when the subset of nodes that must be on tour consists of the entire node set.

A solution to a CSP example is shown in Figure 4.

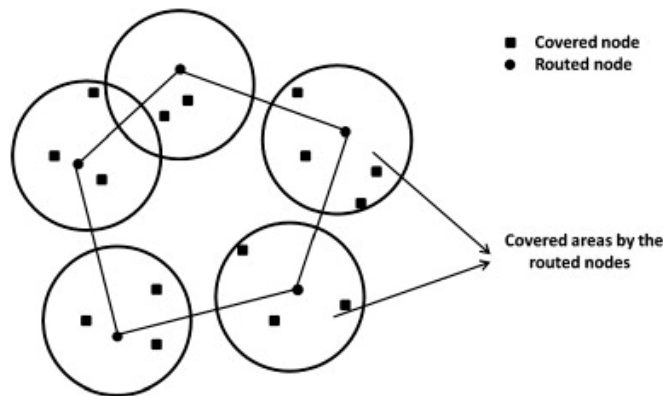


Figure 4: Solution for a CSP [Salari *et al.*, 2012].

2.4.4 Single Vehicle Routing Allocation Problem

Vogt *et al.* [2007] presented a Single Vehicle Routing Allocation Problem (SVRAP) in which a variant of the SVRAP generalizes into the CTP. In the SVRAP, there is a single vehicle together with a set of customers, and the problem is one of deciding a route for the vehicle (starting and ending at given locations) such that it visits some of the customers. In contrast to the usual VRP, not all of the customers need to be visited. Customers not visited by the vehicle can either be allocated to a customer on the vehicle route, or they can be isolated. In addition to the tour routing costs, nodes covered by the tour incur an allocation cost, and nodes not covered by the tour incur a penalty cost. The objective is to minimize a weighted sum of routing, allocation and isolation costs. One special case of the general SVRAP is the CTP when the penalty costs are set high and the allocation costs are set to zero.

For the SVRAP model to generalize into a CTP, the set of vertices $v_h \in T$ that must be on the tour are allocated to a set F_{on} , where $\{0\} \in F_{on}$ and $F_{on} \in V$. The set of vertices $v_l \in W$ that must be covered are allocated to a set F_{off} , where the vertices are off tour but are within the pre-determined distance c from an on tour vertex. Then the allocation cost for vertex i (on tour) to cover vertex j (off tour), d_{ij} , is 0 if vertex j is within distance c from vertex i . Otherwise, d_{ij} is large.

2.5 In-transit Vigilant Covering Tour Problem

The next chapter introduces and discusses a new variant of the CTP for a generic base security defense scenario. This variant, called the in-transit Vigilant CTP (VCTP) model, considers coverage with both vertices and edges. Table 1 summarizes the

characteristics of the various NP-hard problems (TSP, VRP, CSP and CTP) and some of their variants discussed.

Problem	Objective function	Vertices in tour route	Types of vertices	Coverage	No. of vehicles
TSP / mTSP	Minimize distance	All	Single	No	1 / m
GTSP	Minimize distance	Exactly 1 vertex from each cluster	Partitioned into clusters	No	1
VRP	Minimize distance	All	Single with demands	No	m with capacities
CSP	Minimize distance	Unconstrained	Single	Yes by vertices only	1
CTP / mCTP	Minimize distance while covering W	Unconstrained in V All in T	V can be visited T must be visited W must be covered	Yes by vertices only	1 / m
GCTP	Minimize distance while covering W	Unconstrained	V can be visited W must be covered	Yes by vertices only	1
Bi-objective CTP	Minimize distance while covering W & minimize coverage distance	Unconstrained in V All in T	V can be visited T must be visited W must be covered	Yes by vertices only	1
SVRAP	Minimize weighted sum of distance, allocation & isolation cost	All in F_{on}	F_{on} must be visited F_{off} must be covered	Yes by vertices only	1
VCTP / mVCTP	Minimize distance while covering W	Unconstrained in V	V can be visited W must be covered	Yes by both vertices and edges	1 / m

Table 1: Characteristics of related problems.

III. Journal Article

3.1 Introduction

The Maximization of Observability in Navigation for Autonomous Robotic Control (MONARC) project within the Air Force Institute of Technology (AFIT) has an overarching goal of the development of an autonomous robotic, network-enabled, Search, Track, ID, Geo-locate, and Destroy (Kill Chain) capability which would be effective in any environment, at any time.

One area of interest in the MONARC project is mission planning for base security protection of Key Installations (KIN) from adversarial intrusions using autonomous Unmanned Ground Vehicles (UGVs). This UGV mission planning task is multifaceted and requires the consolidation of intelligence, management of system readiness, centralized operational planning and dissemination of Command and Control (C2) information. Sensory data from different locations around the KINs are fused into a Recognized Ground Situation Picture (RGSP) and augmented with intelligence from various agencies. A centralized C2 center consolidates and manages the real-time system serviceability and readiness state of the UGVs. The mission planners input the security requirements, such as key surveillance points, potential intrusion spots, Rules of Engagement (ROE), etc into a Ground Mission Planning System which provides a surveillance approach for use by the team of UGVs.

The protection of a large KIN, such as a military airbase, requires a team of UGVs patrolling along certain routes to effectively cover numerous intrusion spots. The surveillance approach of the security defense task can be formulated as a combinatorial

optimization model, in which multiple security entities must visit multiple locations, while covering certain adversarial locations, in the shortest distance possible traveled by all entities. When adversarial locations are sensed by UGVs at their route location, we can model the problem as a Covering Tour Problem (CTP) [Current, 1981, Current *et al.*, 1989 and Grendreau *et al.*, 1995]; the CTP is a Traveling Salesman Problem (TSP) with Set Covering Problem (SCP) structure. The multiple vehicle variant is a natural extension. Not addressed in prior research, but quite applicable in the current context, is the covering capability while a vehicle is transiting via edges between route locations. This in-transit vigilance component is important to the stated mission planning environment. A new variant of the multiple vehicle Covering Tour Problem (mCTP) model called the in-transit Vigilant CTP (VCTP) is developed and evaluated to meet this requirement as a mission planning tool, applicable to the base security problem.

3.2 Background

The CTP model has been applied extensively in the health care industry, especially for the deployment of mobile health care units traveling in developing countries [Hodgson *et al.*, 1998]. Mobile health care units have access to a limited number of villages due to factors such as infrastructure restrictions, unit capacity and cost. Therefore, it is not feasible to travel to all villages. Instead, a tour route is planned so that the unvisited villages are within reasonable walking distance of the visited villages thereby allowing the needy health care when the health care units visit. The vehicle routes of the health care units are efficiently planned to reduce the amount of travel required, but to enough villages to provide sufficient overall medical coverage. A

real-life problem associated with the planning of mobile health care units was in the Suhum district, Ghana [Oppong *et al.*, 1994] and solved by Hachicha *et al.* [2000] as a CTP.

Another important application of the CTP is the placement of post box locations to reduce the traveling distance of the postal delivery service while ensuring maximum coverage [Labbé *et al.*, 1986]. Good locations of post boxes to cover a region of users and an optimal route for mail distribution are constructed. Alternatively, this approach can be applied to the management of centralized post offices, i.e. post offices are centralized at towns with larger populations while the smaller towns nearby are covered by the centralized post offices.

The CTP model has been applied to the transportation industry such as in the design of bi-level, hierarchical transportation networks [Current *et al.*, 1994]. For an overnight mail delivery service provider such as DHL, Fedex etc, the optimal tour route represents the route taken by the primary vehicle (aircraft) to the distribution centers and the coverage radius is represented by the maximum distance travelled by the delivery trucks from the distribution centers to its customers. This ensures that the overall distribution cost is minimized and provides the required delivery service to its customers.

The mCTP [Hachicha *et al.*, 2000] is defined as a complete undirected graph $G = (V \cup W, E)$ where $V \cup W$ is the vertex set, and $E = \{(v_i, v_j) | v_i, v_j \in V \cup W, i \neq j\}$ is the edge set. Vertex v_0 is a depot (base station), V is the set of vertices that can be visited, $T \subseteq V$ is the set of vertices that must be visited ($v_0 \in T$), and W is the set of vertices (or targets) that must be covered. A distance matrix $C = (c_{ij})$, which satisfies the triangle inequality, indicates the edge length between all vertices ($V \cup W$) and is defined

for E . A final parameter is c , the pre-defined maximum size of the cover. A solution to the mCTP consists in defining a set of m vehicle routes of minimum total length, all starting and ending at the depot such that every vertex in W is covered, subject to some side constraints. Coverage of a vertex is satisfied if it lies within the pre-defined distance c from a vertex in V that belongs to a tour route. Such problems may be infeasible if no element of V covers an element of W .

3.3 CTP for UGV Coverage

One of the defined MONARC scenarios relates to the protection of KINs from potential adversarial intrusions. A team of UGVs are tasked to protect a critical installation. Their surveillance capabilities are augmented by static sensors located throughout the installation. Therefore, a RGSP is available to a mission planner to assist in finding UGV tour routes. The UGVs should only patrol routes that cover all the required checkpoints and the overall route length should be minimized. All UGVs originate from a base station, the depot. There are certain checkpoints that the UGVs must visit (these checkpoints are usually critical ones requiring compulsory surveillance) and there are also checkpoints that may be visited. There are also potential spots where the adversary may appear and these spots must be covered by visiting a checkpoint that is within a fixed proximity distance. Each checkpoint is visited by a UGV and all UGVs return to the base station.

Coverage of targets by a visited checkpoint is defined as the circular area of a fixed radius, where any vertex within the area is covered by that checkpoint. The circular area of coverage is analogous to the effective range of a weapon or sensor system on-

board the UGVs. Each UGV is modeled as an individual vehicle travelling on different routes of minimum length tours. During the route, the UGV covers targets and all targets must be covered for a feasible solution to the overall problem.

There are some key assumptions and limitations made in modeling the base defense security scenario as an mCTP. They are as follows:

- a. All UGVs are homogeneous and have equal capabilities in movement and coverage.
- b. UGVs can visit as many vertices and transit as long as required.
- c. UGVs travel in a straight line between vertices.
- d. Potential adversarial spots are known at a specific point of time or are pre-defined and are thus part of the problem structure.

In reality, UGVs, or for that matter any sensor craft, can sense while traveling. Thus, coverage only at vertices is artificially limiting and coverage while in transit between vertices must be considered. The CTP model is extended to include target coverage via traveled edges. This new variant of the CTP for a generic base security defense scenario, which considers *coverage by both vertices and edges*, is described in the next section. The extension of the in-transit Vigilant CTP into the mCTP variant is discussed in the subsequent section.

3.4 The In-transit Vigilant CTP

The CTP can be used to model an UGV assigned to protect a critical installation. However, a scenario may exist in which a potential adversary spot is not covered by any

checkpoints. In this case, the CTP model yields an infeasible solution. Figure 5 illustrates a single vehicle example in which an infeasible solution is achieved.

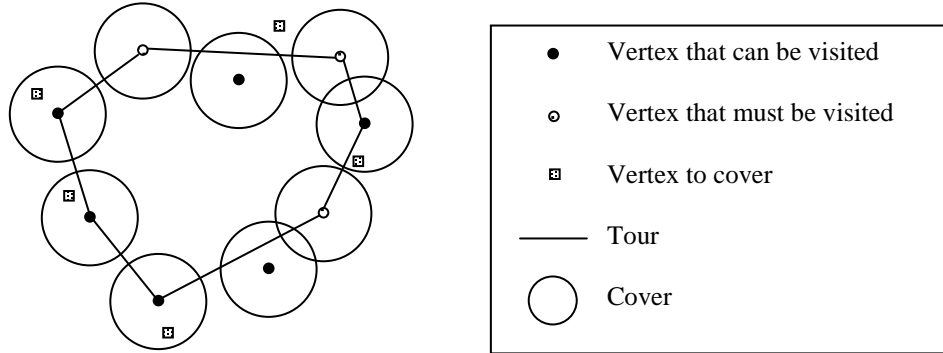
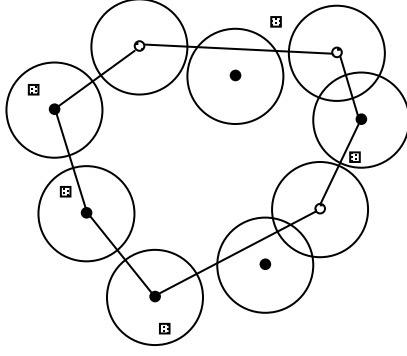


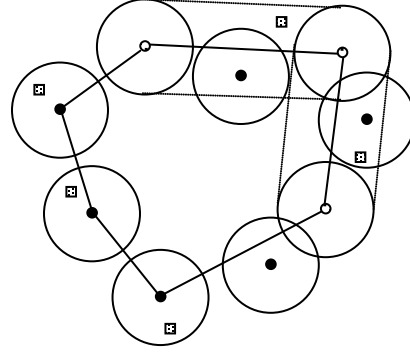
Figure 5: Possible solution for a CTP

The tour in Figure 5 is a minimum length tour constructed with all required vertices visited. However, the solution is infeasible since a visitable vertex to the uppermost vertex to cover does not exist even though the UGV could sense the target while in-transit. The CTP model is modified to allow coverage of such vertices.

Considering coverage *during transit* is a logical assumption for a base security defense problem, i.e. UGVs can cover a potential adversarial spot during its movement between checkpoints. Thus, while an UGV is travelling along the route and transiting between checkpoints, it could pass within some fixed proximity distance and detect (or cover) the adversarial spot. Figure 6 compares the CTP solution from Figure 5 with a solution based on VCTP.



Solution based on CTP model



Solution based on *Vigilant* CTP model

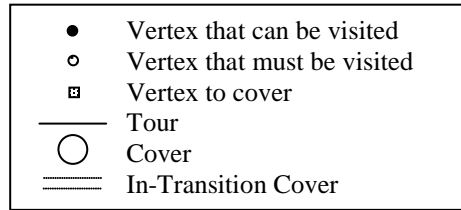


Figure 6: Optimal solution for a CTP and Vigilant CTP.

There are three distinct differences between the models. First, note that the vertex not covered in the previous example is now covered during a route transition with no change of route required. Thus, the revised model effectively increases the amount of coverage as both the traveled vertices and edges provide coverage. Second, we can shorten the tour length, as one of the visited vertices is not required in the VCTP tour since a tour edge provides the requisite coverage. Lastly, the solution based on the Vigilant CTP model is feasible.

The VCTP relaxes some of the model constraints of the CTP via the coverage given by the traveled edges. This effectively sets the *lower-bound* optimal tour length and *upper-bound* target coverage for the CTP formulation and solution.

As illustrated in Figure 6, the coverage of a target by a vertex was changed to coverage by an edge tour length, thus if the same vertices are considered, the solution of

the VCTP model will travel on an equal or lesser number of vertices compared to the CTP model. Since all edge lengths satisfy the triangle inequality, the optimal tour length of the VCTP model will be equal or shorter than the CTP model, forming the lower-bound optimal tour length.

The VCTP model provides a larger coverage area as all vertices and edges lend coverage as compared to coverage via vertices for the CTP model only. The CTP model covers only the circular area around each traveled vertices; however, the VCTP model can cover the circular area and the “thick pencil” area between two traveled vertices. Thus, the VCTP is an upper bound on vertices covered.

3.5 Mathematical Formulation

The mathematical formulation for the VCTP is presented in this section. The basic VRP model [Dantzig *et al.*, 1959] was used as a basis for the VCTP model. We utilize the two-index vehicle flow formulation in the single vehicle variant of the VCTP model [Toth *et al.*, 2002]; it is extended into the three-index vehicle flow formulation for the multiple vehicle VCTP. The two-index vehicle flow formulation which uses $O(n^2)$ binary variables x_{ij} and $O(n)$ binary variables y_i , where x_{ij} and y_i are defined as:

$$x_{ij} = \begin{cases} 1, & \text{edge } (v_i, v_j) \text{ is part of the tour} \\ 0, & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{vertex } v_i \text{ is part of the tour} \\ 0, & \text{otherwise} \end{cases}$$

An important component of the problem formulation lies in the introduction of two pre-processed matrices, α_{ij}^k and β_i^k , which are defined as the in-transit edge coverage and vertex coverage matrices, respectively

For the in-transit edge coverage matrix α_{ij}^k , every vertex that must be covered (k), an i by j matrix is formulated to determine if edge (v_i, v_j) can provide in-transit vigilant coverage against vertex to cover k . Figure 7 illustrates in-transit vigilant coverage of vertex v_k by edge (v_i, v_j) as it lies within the pre-determined perpendicular distance c from edge (v_i, v_j) .

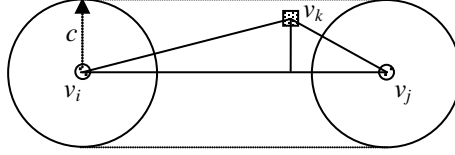


Figure 7: In-transit vigilant coverage by edge (v_i, v_j) on vertex v_k .

Additionally, the construction of the matrix should be carefully considered as a vertex to be covered could lie within the perpendicular distance c from the edge, but fall outside the perpendicular boundaries of edge (v_i, v_j) as shown in Figure 8. The α_{ij}^k will still have a value of 0 in such a case.

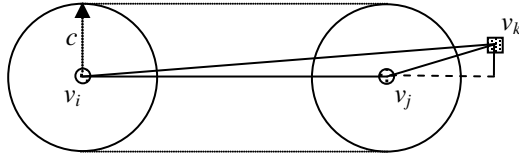


Figure 8: No coverage by edge (v_i, v_j) on vertex v_k .

Thus, if we consider the triangle bounded by vertices v_i , v_j and v_k , the following 2 conditions must hold for vertex v_k to be covered by edge (v_i, v_j) :

1. Vertex v_k must lie within the pre-determined perpendicular distance c from edge (v_i, v_j) .
2. Angles at vertices v_i and v_j must be equal or less than 90 degs.

Therefore the value of α_{ij}^k is defined as:

$$\alpha_{ij}^k = \begin{cases} 1, & \text{edge } (v_i, v_j) \text{ covers } v_k \\ 0, & \text{otherwise} \end{cases}$$

Algorithm 1 defines the construction of the α_{ij}^k matrix:

Given: Set of V vertices and set of W vertices (targets) with their coordinates and distance matrix $C = (c_{ij})$

Set $\alpha_{ij}^k = [0]$ of matrix size V^2W ($V \times V \times W$)

for all i from 1 to V , j from 1 to V ($i \neq j$) and k from 1 to W **do**

Construct triangle with corners i, j & k with corresponding coordinates (x_i, y_i) , (x_j, y_j) & (x_k, y_k)

Let the opposite side lengths be x, y & z where

$$x = c_{jk},$$

$$y = c_{ik},$$

$$z = c_{ij}$$

Let $s = (x + y + z)/2$ where s is the semiperimeter of the triangle

Let $h = \frac{2}{c_{ij}} \sqrt{s(s-x)(s-y)(s-z)}$ where h is the base height of the triangle

if $h \leq c$, **do**

$$\text{Let Angle } i = \arccos\left(\frac{y^2 + z^2 - x^2}{2yz}\right)$$

$$\text{Let Angle } j = \arccos\left(\frac{x^2 + z^2 - y^2}{2xz}\right)$$

if angle $i \leq 90$ degs **AND** angle $j \leq 90$ degs, **then**

$$\alpha_{ij}^k = 1$$

$$\text{else, } \alpha_{ij}^k = 0$$

end

$$\text{else, } \alpha_{ij}^k = 0$$

end

Update α_{ij}^k

end

Algorithm 1: Construction of α_{ij}^k matrix (Edge Covering Matrix).

The vertex covering matrix, β_i^k , is also formulated as a binary matrix. Element (i, k) takes a value of 1 if vertex v_i covers vertex v_k ; vertex v_k lies within the pre-defined Euclidean distance c from vertex v_i .

We now formally define this single vehicle VCTP as an undirected graph $G = (V \cup W, E)$ where $V \cup W$ is the vertex set where $V = \{v_i, v_j\}$ is the set of vertices

that can be visited, $W = \{v_k\}$ is the set of targets that must be covered and $E = \{(v_i, v_j) | v_i, v_j \in V \cup W, i \neq j\}$ is the edge set. Vertex v_0 is a depot (base station). The distance matrix $C = (c_{ij})$ satisfies the triangle inequality and indicates the edge length for all edges in E . The parameter for the pre-defined maximum size of the cover is c . To prevent the formation of subtours, a Subtour Elimination (STE) constraint is added [Dantzig *et al.*, 1954].

The formulation of the integer linear program of the VCTP model is as follows.

Sets

V	Set of vertices to be visited, indexed by i and j
W	Set of vertices to be covered (targets), indexed by k
E	Set of edges $(v_i, v_j) v_i, v_j \in V \cup W, i \neq j$

Data

c_{ij}	Distance of edge (v_i, v_j)
α_{ij}^k	1 if edge (v_i, v_j) covers vertex v_k , 0 otherwise.
β_i^k	1 if vertex v_i covers vertex v_k , 0 otherwise.

Binary Decision Variables

x_{ij}	1 if edge (v_i, v_j) is part of the tour, 0 otherwise.
y_i	1 if node v_i is part of the tour, 0 otherwise.

$$\text{Minimize } \sum_{(v_i, v_j) \in E} c_{ij} x_{ij} \quad (1)$$

Subject to

$$\sum_{i \in V} x_{ij} = y_j \quad \forall j \in V \quad (2)$$

$$\sum_{i \in V} \beta_i^k y_i + \sum_{(v_i, v_j) \in E} \alpha_{ij}^k x_{ij} \geq 1 \quad \forall k \in W \quad (3)$$

$$\sum_{i \in V} x_{ij} = \sum_{l \in V} x_{jl} \quad \forall j \in V \quad (4)$$

$$\sum_{j=1}^{|V|} x_{1j} = 1, \sum_{i=1}^{|V|} x_{i1} = 1 \quad (5)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1, \quad S \subset V, \quad 2 \leq |S| \leq |V| - 1 \quad (6)$$

The objective function (1) minimizes the tour cost. Constraint (2) sets the vertices that are on tour. Constraint (3) ensures all targets are covered, either by a vertex or during transit of an edge. Constraint (4) balances flow through each vertex. Constraint (5) ensures that the tour start and end at the depot. Constraint (6) presents the subtour elimination constraint.

3.6 Extension to Multiple Vehicle

We next extend the VCTP model into a multiple vehicle VCTP (mVCTP). Similar to the VRP, there are m available identical vehicles based at the depot. The mVCTP involves designing a set of minimum total length vehicle routes satisfying the following constraints:

1. There are at most m vehicle routes and each start and end at the depot, v_0 .
2. Each vertex of V belongs to at most one route.
3. Each vertex of W must be covered by an edge or vertex in the routes.

This formulation explicitly indicates the vehicle that traverses an edge, in order to impose more constraints on the routes and overcome some of the drawbacks associated with the two-index model. We use the *three-index vehicle flow formulation* which uses $O(n^2m)$ binary variables x_{hij} and $O(nm)$ binary variables y_{hi} , where x_{hij} and y_{hi} are as defined:

$$x_{hij} = \begin{cases} 1, & \text{edge } (v_i, v_j) \text{ in tour by vehicle } h \\ 0, & \text{otherwise} \end{cases}$$

$$y_{hi} = \begin{cases} 1, & \text{vertex } v_i \text{ in tour by vehicle } h \\ 0, & \text{otherwise} \end{cases}$$

The formulation of the integer linear program of the mVCTP model is as follows.

Sets and Data

The notation for the sets and data are the same as the VCTP model.

Binary Decision Variables

x_{hij} 1 if edge (v_i, v_j) in tour by vehicle h , 0 otherwise.

y_{hi} 1 if node v_i in tour by vehicle h , 0 otherwise.

$$\text{Minimize } \sum_{h=1}^m \sum_{(v_i, v_j) \in E} c_{ij} x_{hij} \quad (1)$$

Subject to

$$\sum_{h=1}^m \sum_{i \in V} x_{hij} = \sum_{h=1}^m y_{hj} \quad (\forall j \in V) \quad (2)$$

$$\sum_{h=1}^m \sum_{i \in V} \beta_i^k y_{hi} + \sum_{h=1}^m \sum_{(v_i, v_j) \in E} \alpha_{ij}^k x_{hij} \geq 1 \quad (\forall k \in W) \quad (3)$$

$$\sum_{i \in V} x_{hij} = \sum_{l \in V} x_{hjl} \quad (\forall j \in V; h = 1, \dots, m) \quad (4)$$

$$\sum_{j=1}^{|V|} x_{h1j} = 1, \quad \sum_{i=1}^{|V|} x_{hi1} = 1 \quad (h = 1, \dots, m) \quad (5)$$

$$\sum_{i \in S} \sum_{j \in S} x_{hij} \leq |S| - 1, \quad (S \subset V; 2 \leq |S| \leq |V| - 1; h = 1, \dots, m) \quad (6)$$

The objective function and the constraints for the mVCTP are similar to the VCTP model, with the inclusion of indices for the multiple vehicles.

3.7 Empirical Study

The VCTP and the mVCTP provide tour cost and target coverage benefits over the CTP and mCTP. Unfortunately, the benefits will likely come at some computational cost. An empirical study is designed and conducted, focused on examining the benefits and costs of the VCTP approach. For this effort, we focus on exact solutions leaving heuristics search methods for follow-on work.

The integer linear program described was coded in [LINGO] and [Microsoft Excel] and tested on randomly generated test problems. Unlike many combinatorial optimization problems where test data and their accompanying optimal solutions are available on [ORLIB] and [TSPLIB], there is no existing database for CTP models. Thus, our test data were constructed from the various Solomon [1987, 2012] data sets, which are VRP with Time Windows data sets using Euclidean distances between vertices. These

routing data sets are classified into randomly generated data points set R1 and clustered data points set C1. As each data set contains 101 points, we randomly select the data points from the set for our study as the vertices to visit. The vertices to cover (targets) are selected from the remaining data points from the same data set.

For the MONARC area of operations, we classify the potential adversarial locations into random and clustered data points to agree with the test problem structure. For a large battlefield with undefined boundaries, we assume that the adversaries appear in a homogeneous fashion and thus the randomly generated data points provide a good approximation. In a battlefield with some high value assets scattered throughout the area of operations, the threats can be identified into certain clusters and the clustered data set is a reasonable fit. Thus, we can make a reasonable case for using each type of problem.

An unattractive feature of the Sub-Tour Elimination (STE) constraint is the exponential increase in the number of constraints with the number of N points to approximately 2^N constraints. Miller-Tucker-Zemlin (MTZ) [1960] introduced another formulation of the STE constraint which adds n variables to the model, but dramatically decreases the number of constraints to approximately n^2 . However, the Dantzig formulation is much tighter than the MTZ as shown by Nemhauser *et al.* [1988]. Desrochers *et al.* [1991] strengthened the MTZ formulation by lifting the MTZ constraints into facets of the TSP polytope. Thus, the Desrochers STE constraint is implemented in the LINGO code as it provides a good compromise between the number of constraints and their tightness. Additionally, as the motivation of the paper is the validation of the VCTP integer linear model, we will only examine problems of small

data size, hence circumventing the computational concerns with exponential increase of STE constraints and reducing the computational time.

The sets V and W were defined by randomly choosing $|V|$ and $|W|$ points from the first $|V| + |W|$ points, respectively from the Solomon data sets. The first point from each data set is chosen as the depot, vertex v_0 . The c_{ij} coefficients were computed as the Euclidean distance between these points. The value of the covering distance c was arbitrarily chosen with a value of 10. For comparison, the data points occupied an approximate 80×80 grid. The α_{ij}^k and β_i^k matrices were pre-processed with Microsoft Excel and read into LINGO. The LINGO source code is in Appendix A.

Tests were run for various combinations of $|V|$ and $|W|$ on the random and clustered data sets. Specifically, the following values were tested: $|V| = 20, 30$, $|W| = 5, 10$. The CTP and VCTP model were used to examine their robustness on the varied data sets. The single vehicle model variants were employed to reduce computational effort and enable better identification of abnormalities. The test runs were executed for 2^4 levels, i.e. $|V|$, $|W|$, random/clustered data points and CTP/VCTP models, for 10 random data sets each. Thus, a total of 160 data sets were run. The test data sets appear in Appendix B.

For comparison, the most optimal tour taken, tour length and computational effort were recorded for both models. The number of targets covered by vertices and edges were also collected for the VCTP model.

The results are summarized in Table 2 to 5 with the headings are defined as:

Tour length:	Optimal tour length;
Iterations:	Number of iterations required by LINGO 11.0;
Tour vertices:	Number of vertices visited (including depot);

Vertex coverage: Number of targets covered by vertices;

Edge coverage: Number of targets covered by edges.

Table 2: Results for $|V| = 20$ and $|W| = 5$.

Problem	CTP model			VCTP model				
	Tour length	Tour vertices	Iterations	Tour length	Tour vertices	Iterations	Vertex coverage	Edge coverage
R101	124.566	4	65,062	124.566	4	87,032	5	0
R102	Infeasible	-	-	118.6011	5	51,037	2	3
R103	Infeasible	-	-	85.16839	3	14,520	1	4
R104	Infeasible	-	-	133.1632	5	162,410	3	2
R105	177.1577	5	120,020	176.0813	5	178,926	4	1
R106	140.7824	5	1,783	136.6277	5	7,233	4	1
R107	Infeasible	-	-	134.9327	5	118,764	3	2
R108	Infeasible	-	-	121.5904	5	71,216	3	2
R109	Infeasible	-	-	Infeasible	-	-	-	-
R110	Infeasible	-	-	139.4547	6	28,749	4	1
R1 avg	147.50	4.67	62,288	145.76	4.78	79,987	3.22	1.78
C101	153.886	5	143,137	153.6151	4	100,712	4	1
C102	Infeasible	-	-	120.069	3	2,708	1	4
C103	192.1403	5	16,689	192.0509	4	31,158	4	1
C104	177.3902	6	159,729	173.2213	5	45,544	3	2
C105	170.9239	5	238,430	162.565	4	489,711	4	1
C106	88.36234	4	10,351	85.09786	4	44,258	4	1
C107	Infeasible	-	-	179.8654	5	29,998	3	2
C108	Infeasible	-	-	Infeasible	-	-	-	-
C109	Infeasible	-	-	Infeasible	-	-	-	-
C110	143.7321	4	114,153	143.7321	4	114,153	5	0
C1 avg	154.41	4.83	113,748	151.71	4.13	107,280	3.50	1.50

Table 3: Results for $|V| = 20$ and $|W| = 10$.

Problem	CTP model			VCTP model				
	Tour length	Tour vertices	Iterations	Tour length	Tour vertices	Iterations	Vertex coverage	Edge coverage
R101	133.0123	7	162,318	127.0204	5	46,268	8	2
R102	Infeasible	-	-	157.3024	5	6,687	5	5
R103	Infeasible	-	-	184.0486	6	93,927	6	4
R104	Infeasible	-	-	162.241	5	363,884	4	6
R105	Infeasible	-	-	Infeasible	-	-	-	-
R106	Infeasible	-	-	182.347	6	9,322	8	2
R107	Infeasible	-	-	213.2143	8	55,003	5	5
R108	Infeasible	-	-	186.6751	5	100,927	4	6
R109	Infeasible	-	-	Infeasible	-	-	-	-
R110	Infeasible	-	-	148.0407	6	11,094	8	2
R1 avg	133.01	7.00	162,318	127.02	5.75	85,889	6.00	4.00
C101	220.2951	7	6,131	216.273	5	7,416	7	3
C102	Infeasible	-	-	227.401	6	141,213	3	5
C103	229.1829	7	120,279	223.3114	5	49,836	7	3
C104	221.4393	7	225,792	214.0772	5	206,575	5	5
C105	Infeasible	-	-	170.9239	5	67,945	7	3
C106	Infeasible	-	-	192.1624	7	35,178	7	3
C107	Infeasible	-	-	179.8654	5	80,666	5	5
C108	Infeasible	-	-	Infeasible	-	-	-	-
C109	Infeasible	-	-	Infeasible	-	-	-	-
C110	Infeasible	-	-	Infeasible	-	-	-	-
C1 avg	223.64	7.00	117,401	217.89	5.43	84,118	5.86	3.86

Table 4: Results for $|V| = 30$ and $|W| = 5$.

Problem	CTP model			VCTP model				
	Tour length	Tour vertices	Iterations	Tour length	Tour vertices	Iterations	Vertex coverage	Edge coverage
R101	124.566	4	2,110,132	124.566	4	4,582,202	5	0
R102	81.31466	4	71,233	81.31466	4	85,620	5	0
R103	Infeasible	-	-	85.16839	3	90,267	1	4
R104	116.5963	6	273,980	115.5935	5	457,159	4	1
R105	165.3386	5	8,075,691	165.3386	5	24,509,374	5	0
R106	140.679	5	65,905	136.6277	5	185,666	4	1
R107	Infeasible	-	-	117.1561	5	141,753	2	3
R108	117.1231	5	4,815,096	117.1231	5	11,537,324	5	0
R109	Infeasible	-	-	102.6575	5	341,768	2	3
R110	147.754	5	1,632,075	136.8977	6	5,601,861	3	2
R1 avg	127.62	4.86	2,434,873	125.35	4.70	4,753,299	3.60	1.40
C101	150.652	5	12,639,563	150.652	5	12,496,739	5	0
C102	116.0504	5	71,723	109.0399	4	188,004	3	2
C103	191.3442	6	10,465,191	191.3354	4	16,372,338	4	1
C104	174.7859	6	21,694,919	173.1091	5	20,295,879	3	2
C105	170.9239	5	22,827,889	162.565	4	16,574,573	4	1
C106	72.82747	3	238,103	72.82747	3	400,078	5	0
C107	Infeasible	-	-	175.747	5	4,769,732	2	3
C108	163.7361	6	6,327,886	163.7307	5	5,056,208	4	1
C109	171.6362	6	29,558,883	166.9823	3	35,535,858	2	3
C110	120.7869	4	75,859	120.7869	4	120,336	5	0
C1 avg	148.08	5.11	11,544,446	145.67	4.20	11,180,975	3.70	1.30

Table 5: Results for $|V| = 30$ and $|W| = 10$.

Problem	CTP model			VCTP model				
	Tour length	Tour vertices	Iterations	Tour length	Tour vertices	Iterations	Vertex coverage	Edge coverage
R101	132.5805	7	120,200	127.0204	5	262,229	8	2
R102	160.3054	8	138,887	157.3024	5	352,454	5	5
R103	Infeasible	-	-	178.5465	6	461,902	6	4
R104	177.7494	10	35,179,360	162.241	5	37,613,680	4	6
R105	Infeasible	-	-	183.5834	5	1,435,766	5	5
R106	162.0383	7	185,151	159.0077	6	81,644	7	3
R107	Infeasible	-	-	189.0305	8	411,934	5	5
R108	138.9221	8	12,843,343	134.9712	7	32,451,923	8	2
R109	Infeasible	-	-	132.0172	6	357,727	7	3
R110	161.7819	9	294,380	148.0407	7	390,280	8	2
R1 avg	155.56	8.17	8,126,887	148.10	6.00	7,381,954	6.30	3.70
C101	215.9099	7	8,112,801	212.0746	5	17,819,282	7	3
C102	216.4693	8	4,427,765	204.0802	6	1,556,739	5	5
C103	226.3285	7	12,494,718	220.4994	5	8,249,504	7	3
C104	218.2831	7	69,249,875	214.0772	5	78,678,647	5	5
C105	Infeasible	-	-	170.9239	5	2,965,931	7	3
C106	Infeasible	-	-	170.5045	5	2,292,541	7	3
C107	Infeasible	-	-	175.747	5	9,761,083	4	6
C108	163.9464	7	2,493,496	163.7307	5	5,356,536	5	5
C109	207.3529	8	55,206,267	194.5455	5	25,891,461	3	7
C110	180.1757	7	2,300,252	177.4336	4	7,855,860	5	5
C1 avg	204.07	7.29	22,040,739	198.06	5.00	16,042,758	5.50	4.50

We notice the following general observations from Tables 2 to 5:

1. For data sets which yielded feasible solutions for both CTP and VCTP models, the VCTP tours are always shorter.
2. The vertex coverage dominates but edge coverage is utilized for VCTP models.
3. Many more problems have feasible solutions when edge coverage is exploited.

Graphical examples comparing tour solutions from the CTP and VCTP models, for data sets C101 and R101, are shown in Figure 9 and 10, respectively.

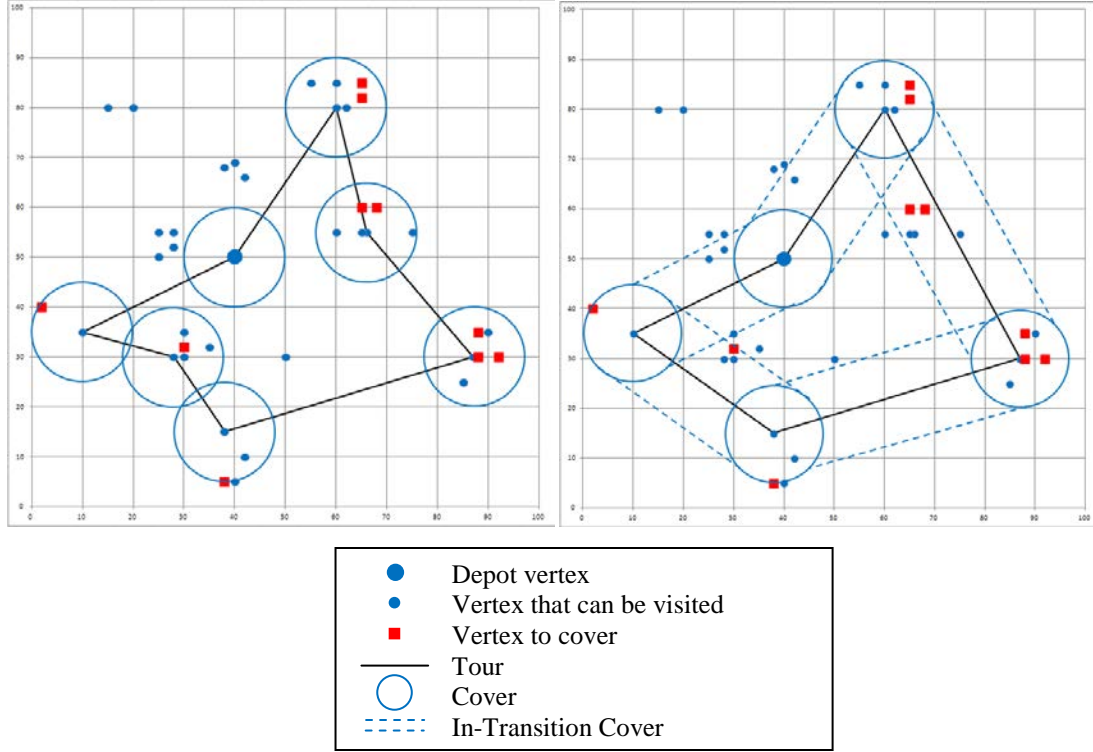


Figure 9: Comparison of CTP (left) and VCTP (right) solution for data set C101.

Figure 9 provides a scatter plot of cluster data set C101 with $|V| = 30$, $|W| = 10$ with solutions from the CTP and VCTP model that clearly illustrate the difference in coverage. We observe that the CTP model requires 7 traveled vertices and an overall longer tour length to cover all targets. The VCTP model requires only 5 vertices and 2 edges for full coverage with a shorter tour length.

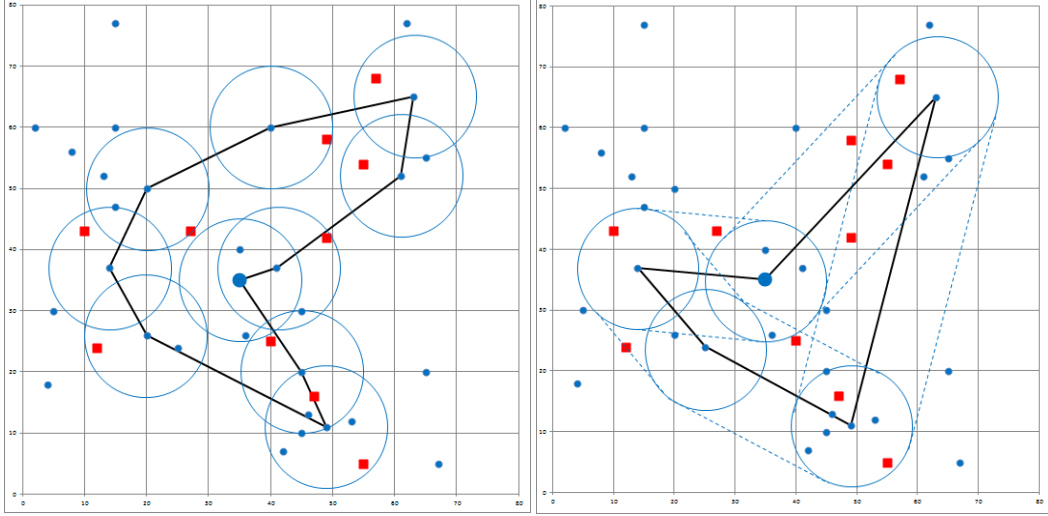


Figure 10: Comparison of CTP (left) and VCTP (right) solution for data set R101.

For the random data set R101, with $|V| = 30$, $|W| = 10$, we see a significant difference in the optimal tour route between the CTP and VCTP model. The original model requires 10 traveled vertices to cover all 10 targets. The VCTP model utilizes 5 vertices and all 5 edges for target coverage.

3.8 Analysis of Results

From the raw results, we calculated the following Measures of Performance (MOPs) to further compare the performance of both models:

1. Number of times the Vigilant CTP model has a shorter tour length than the original CTP model.
2. Average percentage tour length savings.
3. Average percentage of targets covered by edges.
4. Computational efficiency (in number of iterations).

Generally, the VCTP model performed better than the CTP model in an operational sense not considering the computational burden. The number of infeasible solutions for each combination of test runs is shown in Table 6.

Table 6: Number of infeasible solutions for each combination of data sets.

Problem	V	W	CTP model	VCTP model
R1	20	5	7	1
C1			4	2
R1		10	9	2
C1			7	3
R1	30	5	3	0
C1			1	0
R1		10	4	0
C1			3	0
Total			38	8

Of the 80 problems, there were only 8 infeasible problems using the VCTP model. However, there were 38 infeasible problems using the CTP model. The main bulk of infeasibilities occurred for the $|V| = 20$, $|W| = 10$ scenario which is reasonable as the number of available vertices for travel may not be sufficient to cover the proportionally large number of targets, given the fixed coverage distance.

We also compared results, based on the optimal tour length generated. A shorter tour length equates to better performance. The raw results showed that the VCTP model performed better in 63 instances than the CTP model. The number of times of VCTP superior performance, identical performance and infeasibilities of the two models is tabulated in Table 7.

Table 7: Comparison of performance between the CTP and VCTP models.

Problem	V	W	Better performance	Identical performance	Infeasibilities on both models
R1	20	5	8	1	1
C1			7	1	2
R1		10	8	0	2
C1			7	0	3
R1	30	5	6	4	0
C1			7	3	0
R1		10	10	0	0
C1			10	0	0
Total			63	9	8

A non-parametric binomial test is conducted with the null hypothesis of same performance by both models and alternate hypothesis of difference performance based on the Table 7 results.

$$H_0: Performance_{CTP} = Performance_{VCTP}$$

$$H_1: Performance_{CTP} \neq Performance_{VCTP}$$

For $\alpha = 0.01$, the critical region lies outside the confidence interval of $72 \left(\frac{1}{2}\right) \pm 2.326 \sqrt{72 \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)} = [26.1, 45.9]$; we see 63 instances of better performance meaning the null hypothesis is rejected at the 99% significant level and we conclude that the VCTP model performs better.

We also observed that the VCTP model performed better as the number of targets increases. This is attributed to the complimentary capability of vertex and edge coverage; as the CTP is unable to cover as many targets based on vertex coverage alone.

The average tour length for each solved problem was computed to compare the CTP and VCTP performance. The average tour length of the original CTP model was used as the basis and Table 8 shows the percentage of average tour length savings.

Table 8: Percentage of savings in average tour lengths.

Problem	V	W	Average tour length savings (in %)
R1	20	5	1.18
C1			1.74
R1		10	4.50
C1			2.57
R1	30	5	1.78
C1			1.63
R1		10	4.80
C1			2.94
Total			2.64

The overall average tour length savings was 2.64%. For each problem set, we observed that the optimal tour length of the VCTP model is always less than or equal to the original CTP model. This agrees with our claim that the VCTP sets the lower-bound tour length for the original CTP solution. Again, we observe that the VCTP model yielded a higher percentage of average tour length savings when the number of targets was higher.

The next MOP examines the improvement in coverage due to the edge covering capability of the VCTP model. We calculated the percentage of targets covered by edges. The results are tabulated in Table 9.

Table 9: Percentage of targets covered by edges in the VCTP model.

Problem	$ V $	$ W $	Average number of targets covered by edges (in %)
R1	20	5	35.6
C1			30.0
R1		10	40.0
C1			39.7
R1	30	5	28.0
C1			26.0
R1		10	37.0
C1			45.0
Total			35.2

The overall percentage of targets covered by edges is significant at 35.2%. In the presence of higher number of targets, the VCTP model provided more coverage via the edges. As expected, the edge covering capability is exploited by the route construction.

Computational efficiency of the two models was compared using the number of iterations (branch and bound nodes) required to generate the optimal tour length in LINGO 11.0. Only instances where both the original CTP and VCTP model gave feasible solutions were compared. The percentage of computational efficiency was calculated as the difference in the number of iterations between the two models divided by the number of iterations used by CTP. The overall percentage in computational efficiency is a weighted average based on the number of instances across each problem set. The comparison of the computational efficiency is shown in Table 10.

Table 10: Number of iterations by both models and percentage comparison.

Problem	/V/	/W/	Average number of iterations		Improvement of computational efficiency (in %)
			CTP model	VCTP model	
R1	20	5	62,288	91,064	- 46.20
C1			113,748	137,589	- 20.96
R1		10	162,318	46,268	71.50
C1			117,401	87,942	25.09
R1	30	5	2,434,873	6,708,458	- 175.52
C1			11,544,446	11,893,335	- 3.02
R1		10	8,126,887	11,858,702	- 45.92
C1			22,040,739	20,772,576	5.75
Overall					- 38.30

The VCTP model requires significantly more computational effort to generate optimal tour lengths. This is expected as the VCTP model uses V^2W more variables due to the additional α_{ij}^k related variables for edge coverage computation. For comparison, in

a $|V| = 30$, $|W| = 10$ scenario, there are 970 and 9970 variables in CTP and VCTP, respectively.

3.9 Other extensions

The VCTP is a baseline model for a generic base security defense scenario. It is natural to discuss other extensions to improve the computational efficiency and the problem formulation. Some areas for further research are as listed:

1. Heuristic Method. The VCTP is NP-hard. Furthermore, our results show the problems are hard in practice. A heuristic is a polynomial time algorithm that produces optimal or near optimal solutions on some input instances [Feige, 2005]. For a relatively small instance of $|V| = 30$, $|W| = 10$, we observed that 78 million iterations by LINGO 11.0 are required. For a real-world scenario where $|V|$ could go quite large, exact solvers are impractical and heuristic methods should be developed. As the VCTP is a TSP with SCP structure, a potential heuristic approach is the combination of the GENIUS heuristic [Gendreau *et al.*, 1992] for TSP with a modified version of the PRIMAL1 set covering heuristic [Balas *et al.*, 1980] to account for the additional edge coverage capability.
2. Multiple vehicle variant. The TSP has received a lot of research attention; the multiple TSP is more adequate to model real-world applications [Bektas, 2006]. Similarly, further research should be conducted on the multiple vehicle variant of the VCTP model.
3. Dynamic Routing. Based on the assumptions made for the VCTP, it is a *static* and *deterministic* problem, where all inputs are known beforehand and routes do not change during execution [Pillac, 2011]. Real-world applications often include two important

dimensions: evolution and quality of information. Evolution implies that information may change during execution of routes and quality reflects possible uncertainty on the available data. Thus, for a *dynamic* and *stochastic* VCTP, the tour route can be redefined in an ongoing fashion based on changing travel vertices and appearance of pop-up targets.

IV. Conclusions

4.1 Contributions

An application of the newly defined VCTP is used to model an UGV assigned to base security protection. The VCTP has the novel additional edge coverage, to model the sensing capability of the UGV while traveling.

The empirical study showed that the VCTP model performed better across all combinations of scenarios. Specifically, it performed significantly better when more targets need to be covered. For the same problem data sets, the VCTP is more robust yielding more feasible solutions (72 out of 80) as compared to the CTP (42 out of 80). All VCTP optimal tour lengths were also equal or shorter than the CTP model, with an average tour length savings of 2.64%. The edge coverage capability of the VCTP accounted for 35.2% of target coverage. However, the VCTP required 38.3% more computational effort for tour length generation.

The main contribution of this thesis is a nascent combinatorial optimization model for routing UGVs while highlighting the importance and usefulness of both vertex and edge coverage. This model can be utilized as a first cut mission planning optimizer tool to address the MONARC base security problem.

4.2 Future Work

The immediate focus for future research is the development of a high quality and quick running heuristic solver. As real-world problems are typically large sized, exact solvers are inadequate as they are computationally expensive and require significant running time. Additionally, a force protection scenario may evolve quickly with changes

in the routing points or appearance of adversaries. Thus, a fast heuristic solver to provide real-time updates of near-optimal routes is important and usually sufficient. This reinforces the need to complement the exact solver in the provision of a holistic mission planning tool with preliminary optimal and pseudo-dynamic near-optimal routing capabilities.

The flexibility of the TSP formulation with dual set covering structure allows customization for different applications. Additional indices and side constraints can be included for multiple vehicle and other unique variants respectively to better mimic real-world situations. For example, some UGVs have better sensory coverage when static (at a vertex) than during transit (along an edge). The edge and vertex coverage matrices (α_{ij}^k and β_i^k) are therefore modified accordingly.

Appendix A. LINGO source code

LINGO source code for VCTP for $|V| = 30$ and $|W| = 10$

```
model:

sets:
Vertex / 1..30 /: V; !Number/Sequence of vertices/nodes;
Target / 1..10 /: T; !Number of targets to be covered;
Link1 (Vertex, Vertex): C, X; !where C is the distance
matrix and X(i,j) = 1 if arc i to j is part of tour;
Node (Vertex): Y; !where Y(i) = 1 if node i is part of
tour;
Link2 (Target, Vertex, Vertex): alpha; !where alpha(k,i,j)
= 1 if arc(i,j) covers node k;
Link3 (Target, Vertex): beta; !where beta(k,i) = 1 if node
i covers node k;

endsets

data:
C = @ole('\filename.xlsx','Matrix'); !Distance matrix from
excel;
alpha = @ole('\filename.xlsx','alpha'); !Pre-processed arc
covering matrix from excel;
beta = @ole('\filename.xlsx','beta'); !Pre-processed matrix
to be loaded from excel;

@text() = @writefor( Link1(i,j)|X(i,j) #EQ# 1:
'Route from ',i, ' to ',j, @newline(1)); !Output optimal
route taken;
@text() = @writefor( Node(i)|Y(i) #EQ# 1:
'Vertex ',i,' used', @newline(1)); !Output vertices
traveled on optimal route;

enddata

min = @sum(Link1: C * X);
!Objective function is to minimize the length of tour;

@for(Vertex(j):
@sum(Vertex(i)|i #NE# j: X(i,j)) = Y(j);
!To set which nodes are on tour (Linking constraint to
ensure if node(j) is on tour, there is EXACTLY 1 arc that
flows in);
```

```

);

@for(Target(k):
@sum(Link3(k,i): beta * Y) + @sum(Link2(k,i,j): alpha * X)
> 1;
!To ensure that all targets are covered by either a node or
transit of an arc;
);

@for(Vertex(j):
@sum(Vertex(i)|i #NE# j: X(i,j)) - @sum(Vertex(l)|j #NE# l:
X(j,l)) = 0;
!To ensure balance flow through all nodes;
);

@sum(Link1(i,j)|i #EQ# 1 #AND# i #NE# j: X) = 1;
@sum(Link1(i,j)|j #EQ# 1 #AND# i #NE# j: X) = 1;
!To ensure that the tour starts from vertex 1 (depot
constraint) and ends at vertex 1;

N = @size(Vertex); !Number of elements in the set;

@for(Vertex(k):
@for(Vertex(j)|j #GT# 1 #AND# j #NE# k: V(j) > V(k) +
X(k,j) - (N-2)*(1 - X(k,j)) + (N-3)*X(j,k));
);
!Miller-Tucker-Zemlin (MTZ) subtour elimination constrains
improved by Descrochers and Laporte (1991);

@for(Link1: @bin(X));      !For binary values of X;
@for(Node: @bin(Y));      !For binary values of Y;

end

```

Appendix B. Data sets

Table 11 and 12 below show the data points for the random set R1 with $|V| = 30$ and 20 respectively. For data sets with $|W| = 5$, the $|W|$ values for S/N 1 to 5 are taken.

Table 11: Data points for random set R1 with $|V| = 30$

	S/N	R101		R102		R103		R104		R105		R106		R107		R108		R109		R110	
		x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
$ V $	1	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35
	2	53	52	49	42	64	42	61	52	20	26	17	34	18	24	50	35	63	23	26	27
	3	50	35	26	35	22	22	20	26	57	48	4	18	6	68	24	12	25	21	30	25
	4	42	7	45	30	2	60	25	24	40	60	37	56	10	20	37	56	37	47	30	60
	5	35	69	18	24	20	40	63	65	55	5	26	27	41	37	11	14	42	7	53	12
	6	20	50	49	73	56	37	15	77	8	56	40	60	35	69	49	11	35	17	20	26
	7	19	21	22	27	18	18	45	10	10	43	5	30	17	34	53	12	15	60	25	24
	8	15	19	26	52	2	48	46	13	20	65	62	77	55	5	47	16	6	38	65	55
	9	25	24	15	19	55	45	45	30	27	43	20	50	63	23	26	52	20	50	55	20
	10	37	56	44	17	5	5	15	47	49	73	63	23	45	30	63	65	62	77	6	68
	11	55	45	37	47	62	77	14	37	41	37	56	39	27	69	57	29	25	24	14	37
	12	63	23	63	65	37	31	65	55	15	47	36	26	37	31	16	22	55	20	53	52
	13	10	43	55	45	27	69	53	12	2	48	25	24	56	39	47	47	55	60	19	21
	14	11	14	40	25	35	40	41	37	28	18	32	12	4	18	22	27	10	43	11	31
	15	61	52	49	58	37	56	49	11	50	35	25	21	15	30	31	52	65	55	63	65
	16	65	20	15	10	12	24	45	20	49	42	40	25	25	30	44	17	55	5	37	56
	17	67	5	63	23	47	16	15	60	49	58	24	12	60	12	5	30	41	37	2	60
	18	2	60	11	31	5	30	13	52	26	35	15	10	11	31	41	37	47	16	63	23
	19	57	68	23	3	53	52	62	77	19	21	8	56	49	42	15	60	28	18	55	60
	20	14	37	12	24	49	42	67	5	21	24	6	38	55	54	20	26	20	20	49	73
	21	53	43	15	77	11	14	36	26	53	12	15	19	36	26	31	67	37	56	47	16
	22	23	3	65	55	53	43	65	20	30	60	42	7	31	52	55	20	45	30	15	60
	23	45	65	20	20	13	52	5	30	22	22	65	55	26	35	2	60	5	30	35	69
	24	26	35	22	22	60	12	2	60	23	3	11	14	45	20	25	24	56	37	15	10
	25	30	60	13	52	15	77	8	56	57	29	53	43	22	27	15	30	32	12	41	49
	26	21	24	42	7	6	38	40	60	20	50	41	49	31	67	35	17	18	24	24	58
	27	26	27	53	12	10	20	4	18	20	40	45	10	64	42	20	40	45	10	6	38
	28	64	42	47	16	20	26	20	50	35	40	45	30	16	22	65	20	5	5	61	52
	29	55	20	18	18	45	65	35	40	11	31	60	12	30	25	25	30	11	31	55	45
	30	45	20	28	18	65	55	42	7	40	25	55	5	10	43	26	35	14	37	22	27
$ W $	1	6	38	53	43	61	52	12	24	55	60	56	37	28	18	14	37	11	14	18	18
	2	55	60	30	60	56	39	47	16	2	60	2	48	46	13	35	40	40	25	57	29
	3	16	22	20	26	50	35	49	58	53	52	49	58	53	12	60	12	30	60	49	58
	4	28	18	56	37	49	58	27	43	60	12	49	42	65	55	32	12	27	43	15	19
	5	63	65	27	43	63	65	10	43	18	24	65	20	18	18	40	60	15	30	13	52
	6	22	27	6	38	20	65	57	68	26	52	57	29	15	77	42	7	15	19	20	40
	7	13	52	55	5	24	58	55	5	11	14	45	20	49	58	36	26	49	58	64	42
	8	49	42	45	10	55	5	55	54	24	12	30	25	65	20	18	24	63	65	57	68
	9	41	49	45	65	37	47	40	25	25	24	61	52	55	60	20	50	41	49	31	67
	10	20	26	8	56	16	22	49	42	12	24	35	40	41	49	27	69	37	31	40	25

Table 12: Data points for random set R1 with $|V| = 20$

	S/N	R101		R102		R103		R104		R105		R106		R107		R108		R109		R110	
		x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
$ V $	1	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35
	2	53	52	63	65	37	31	61	52	20	26	17	34	18	24	50	35	63	23	26	27
	3	50	35	55	45	27	69	20	26	57	48	4	18	6	68	24	12	25	21	30	25
	4	42	7	40	25	35	40	25	24	40	60	37	56	10	20	37	56	37	47	30	60
	5	35	69	49	58	37	56	63	65	55	5	26	27	41	37	11	14	42	7	53	12
	6	20	50	15	10	12	24	15	77	8	56	40	60	35	69	49	11	35	17	20	26
	7	19	21	63	23	47	16	45	10	10	43	5	30	17	34	53	12	15	60	25	24
	8	15	19	11	31	5	30	46	13	20	65	62	77	55	5	47	16	6	38	65	55
	9	25	24	23	3	53	52	45	30	27	43	20	50	63	23	26	52	20	50	55	20
	10	37	56	12	24	49	42	15	47	49	73	63	23	45	30	63	65	62	77	6	68
	11	55	45	15	77	11	14	14	37	41	37	56	39	27	69	57	29	25	24	14	37
	12	63	23	65	55	53	43	65	55	15	47	36	26	37	31	16	22	55	20	53	52
	13	10	43	20	20	13	52	53	12	2	48	25	24	56	39	47	47	55	60	19	21
	14	11	14	22	22	60	12	41	37	28	18	32	12	4	18	22	27	10	43	11	31
	15	61	52	13	52	15	77	49	11	50	35	25	21	15	30	31	52	65	55	63	65
	16	65	20	42	7	6	38	45	20	49	42	40	25	25	30	44	17	55	5	37	56
	17	67	5	53	12	10	20	15	60	49	58	24	12	60	12	5	30	41	37	2	60
	18	2	60	47	16	20	26	13	52	26	35	15	10	11	31	41	37	47	16	63	23
	19	57	68	18	18	45	65	62	77	19	21	8	56	49	42	15	60	28	18	55	60
	20	14	37	28	18	65	55	67	5	21	24	6	38	55	54	20	26	20	20	49	73
$ W $	1	6	38	53	43	61	52	12	24	55	60	56	37	28	18	14	37	11	14	18	18
	2	55	60	30	60	56	39	47	16	2	60	2	48	46	13	35	40	40	25	57	29
	3	16	22	20	26	50	35	49	58	53	52	49	58	53	12	60	12	30	60	49	58
	4	28	18	56	37	49	58	27	43	60	12	49	42	65	55	32	12	27	43	15	19
	5	63	65	27	43	63	65	10	43	18	24	65	20	18	18	40	60	15	30	13	52
	6	22	27	6	38	20	65	57	68	26	52	57	29	15	77	42	7	15	19	20	40
	7	13	52	55	5	24	58	55	5	11	14	45	20	49	58	36	26	49	58	64	42
	8	49	42	45	10	55	5	55	54	24	12	30	25	65	20	18	24	63	65	57	68
	9	41	49	45	65	37	47	40	25	25	24	61	52	55	60	20	50	41	49	31	67
	10	20	26	8	56	16	22	49	42	12	24	35	40	41	49	27	69	37	31	40	25

Table 13 and 14 below show the data points for the clustered set C1 with $|V| = 30$ and 20 respectively. Similarly, for data sets with $|W| = 5$, the $|W|$ values for S/N 1 to 5 are taken.

Table 13: Data points for clustered set C1 with $|V| = 30$

	S/N	C101		C102		C103		C104		C105		C106		C107		C108		C109		C110	
		x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
$ V $	1	40	50	40	50	40	50	40	50	40	50	40	50	40	50	40	50	40	50	40	50
	2	40	5	30	30	70	58	28	35	20	85	90	35	35	5	38	68	20	50	22	85
	3	75	55	42	66	28	55	55	80	28	55	32	30	15	80	25	30	65	85	25	55
	4	60	80	8	45	50	30	15	75	20	55	50	40	2	40	33	32	23	55	35	5
	5	50	30	38	5	48	30	48	40	40	5	28	35	68	60	42	66	25	30	26	32
	6	60	55	10	40	63	58	20	80	0	45	8	40	44	5	30	52	68	60	50	35
	7	87	30	50	35	35	30	50	30	75	55	75	55	53	35	47	35	42	15	30	50
	8	40	69	65	60	88	30	60	60	95	30	40	5	40	69	8	45	30	30	55	80
	9	25	50	33	32	60	55	38	70	35	32	25	30	15	75	65	82	20	85	40	69
	10	20	80	35	66	5	35	8	45	42	15	5	35	85	25	40	69	58	75	20	80
	11	15	80	42	10	65	55	40	69	48	30	42	66	30	32	25	52	35	66	63	58
	12	66	55	28	30	20	50	35	69	87	30	42	10	45	35	40	66	85	25	53	35
	13	42	10	87	30	23	55	30	30	42	66	10	40	87	30	88	30	70	58	42	15
	14	42	66	53	35	62	80	38	15	50	40	50	35	92	30	92	30	5	45	30	32
	15	28	55	25	55	87	30	65	60	45	70	45	70	55	80	35	32	22	75	48	40
	16	38	68	0	40	60	60	65	55	45	35	44	5	38	68	25	50	35	32	38	68
	17	10	35	30	52	40	5	35	32	35	5	20	85	66	55	35	66	48	30	20	85
	18	28	30	38	15	45	35	26	32	45	30	40	69	55	85	53	35	23	52	32	30
	19	65	55	32	30	45	70	45	65	38	5	60	85	20	50	5	45	35	69	20	55
	20	60	85	85	35	67	85	10	40	40	69	58	75	67	85	23	55	60	55	95	35
	21	38	15	23	52	92	30	72	55	88	35	10	35	0	45	60	80	8	45	85	35
	22	28	52	40	15	75	55	33	32	5	35	70	58	95	35	63	58	35	30	66	55
	23	55	85	20	80	30	30	28	55	44	5	45	65	48	40	25	35	62	80	25	50
	24	85	25	48	30	55	80	65	85	55	85	68	60	75	55	38	15	38	68	40	5
	25	90	35	25	30	28	35	60	55	65	82	47	40	28	35	62	80	50	30	40	66
	26	35	32	35	32	60	80	2	40	70	58	62	80	88	30	42	10	15	75	38	5
	27	30	35	60	80	2	40	45	35	55	80	20	80	50	40	22	75	18	75	35	69
	28	62	80	85	25	20	80	88	30	28	52	35	30	60	55	30	50	35	5	8	45
	29	25	55	23	55	40	69	68	60	8	45	48	30	23	52	53	30	63	58	60	85
	30	30	30	20	85	38	15	35	30	25	50	38	15	10	35	35	5	30	35	10	40
$ W $	1	68	60	45	65	25	30	23	55	30	52	45	30	95	30	66	55	25	85	50	40
	2	65	85	38	68	20	85	5	45	38	68	30	30	72	55	40	15	38	5	48	30
	3	65	60	28	52	85	35	87	30	38	15	33	35	40	15	42	68	47	35	90	35
	4	30	32	47	35	22	85	75	55	95	35	30	35	28	52	15	80	25	35	45	65
	5	2	40	22	75	25	50	25	30	30	50	45	68	42	15	23	52	20	55	88	35
	6	88	30	25	50	10	40	22	85	32	30	22	85	65	60	45	68	72	55	28	35
	7	38	5	30	35	10	35	90	35	42	68	63	58	33	35	50	30	45	68	10	35
	8	92	30	95	30	38	70	23	52	66	55	23	52	25	52	68	60	30	52	45	30
	9	88	35	22	85	20	55	25	35	60	55	15	80	30	35	50	35	25	55	87	30
	10	65	82	58	75	25	35	22	75	23	55	2	40	33	32	47	40	42	10	65	60

Table 14: Data points for clustered set C1 with $|V| = 20$

	S/N	C101		C102		C103		C104		C105		C106		C107		C108		C109		C110	
		x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y	x	y
$ V $	1	40	50	40	50	40	50	40	50	40	50	40	50	40	50	40	50	40	50	40	50
	2	40	5	28	30	20	50	28	35	20	85	90	35	35	5	38	68	20	50	22	85
	3	75	55	87	30	23	55	55	80	28	55	32	30	15	80	25	30	65	85	25	55
	4	60	80	53	35	62	80	15	75	20	55	50	40	2	40	33	32	23	55	35	5
	5	50	30	25	55	87	30	48	40	40	5	28	35	68	60	42	66	25	30	26	32
	6	60	55	0	40	60	60	20	80	0	45	8	40	44	5	30	52	68	60	50	35
	7	87	30	30	52	40	5	50	30	75	55	75	55	53	35	47	35	42	15	30	50
	8	40	69	38	15	45	35	60	60	95	30	40	5	40	69	8	45	30	30	55	80
	9	25	50	32	30	45	70	38	70	35	32	25	30	15	75	65	82	20	85	40	69
	10	20	80	85	35	67	85	10	40	42	15	5	35	85	25	40	69	58	75	20	80
	11	15	80	23	52	92	30	72	55	48	30	42	66	30	32	25	52	35	66	63	58
	12	66	55	40	15	75	55	33	32	87	30	42	10	45	35	40	66	85	25	53	35
	13	42	10	20	80	30	30	28	55	42	66	10	40	87	30	88	30	70	58	42	15
	14	42	66	48	30	55	80	65	85	50	40	50	35	92	30	92	30	5	45	30	32
	15	28	55	25	30	28	35	60	55	45	70	45	70	55	80	35	32	22	75	48	40
	16	38	68	35	32	60	80	2	40	45	35	44	5	38	68	25	50	35	32	38	68
	17	10	35	60	80	2	40	45	35	35	5	20	85	66	55	35	66	48	30	20	85
	18	28	30	85	25	20	80	88	30	45	30	40	69	55	85	53	35	23	52	32	30
	19	65	55	23	55	40	69	68	60	38	5	60	85	20	50	5	45	35	69	20	55
	20	60	85	20	85	38	15	35	30	40	69	58	75	67	85	23	55	60	55	95	35
$ W $	1	68	60	45	65	25	30	23	55	30	52	45	30	95	30	66	55	25	85	50	40
	2	65	85	38	68	20	85	5	45	38	68	30	30	72	55	40	15	38	5	48	30
	3	65	60	28	52	85	35	87	30	38	15	33	35	40	15	42	68	47	35	90	35
	4	30	32	47	35	22	85	75	55	95	35	30	35	28	52	15	80	25	35	45	65
	5	2	40	22	75	25	50	25	30	30	50	45	68	42	15	23	52	20	55	88	35
	6	88	30	25	50	10	40	22	85	32	30	22	85	65	60	45	68	72	55	28	35
	7	38	5	30	35	10	35	90	35	42	68	63	58	33	35	50	30	45	68	10	35
	8	92	30	95	30	38	70	23	52	66	55	23	52	25	52	68	60	30	52	45	30
	9	88	35	22	85	20	55	25	35	60	55	15	80	30	35	50	35	25	55	87	30
	10	65	82	58	75	25	35	22	75	23	55	2	40	33	32	47	40	42	10	65	60

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Vita

Major Huang Teng Tan enlisted in the Singapore Armed Forces in Dec 1997. After Basic Military Training, he was assigned to Officer Cadet School (OCS) for his professional training as an Air Warfare Officer (Ground Based Air Defence). He graduated from OCS and was commissioned in Apr 1999.

His first assignment was at 18th Divisional Air Defence Artillery Battalion (18 DA Bn) as a MISTRAL Fire Unit Commander. In Oct 2000, Major Tan was assigned to Imperial College London, United Kingdom, to earn a Bachelor of Engineering Degree in Mechanical Engineering.

Upon graduation, he was assigned to numerous staff and command appointments. Notably, Major Tan served as a Research & Technology Officer in the Defense Research & Technology Office of the Ministry of Defense and as a Battery Commander in 18 DA Bn.

In Mar 2011, he embarked on an 18-months double Masters program jointly conducted by the National University of Singapore and Air Force Institute of Technology. Upon graduation, Major Tan will be assigned to Air Plans Department, HQ Republic of Singapore Air Force.

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